

# Higgs Masses and $CP$ Violation in SUSY Models

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## Abstract

Though the mass of Higgs particle is the parameter determined by the experiment in the standard model (SM), SUSY models have rather predictive power for the lightest Higgs mass, and its upper bound in some SUSY models are close to the observable region in LEP2. The upper bound of the lightest Higgs mass is analysed systematically on the basis of the  $CP$  violation in the minimal and the next minimal supersymmetric standard model (MSSM and NMSSM). In the explicit  $CP$  violation case, the mass bound is large around  $130 \sim 160$  GeV in both models. In the spontaneous  $CP$  violation case induced by the radiative effects, the lightest Higgs mass upper bound is about 52 GeV and sum of two light neutral Higgs should be around  $O(100)$  GeV in the NMSSM in contrast to the one in the MSSM which implies about 6 GeV. This model gives the interesting predictions for the neutron electric dipole moment.

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# 1 Introduction

The  $CP$  physics is one of the most exciting topics in the recent particle physics. The origin of  $CP$  violation is still in a mystery. In the standard model (SM), the origin of  $CP$  phase exists in Kobayashi-Maskawa (KM) matrix[1]. In the SM,  $CP$  is violated through the yukawa coupling and is conserved in the Higgs potential. The SM Lagrangian is not invariant under the  $CP$  transformation and  $CP$  is violated explicitly. However, if the Higgs sector is extended into the one with two or more doublets, we have richer  $CP$  violation sources. In the multi-Higgs models,  $CP$  is generally violated explicitly and/or spontaneously in the Higgs potential[2][3]. In the spontaneous  $CP$  violation, vacuum expectation values (VEVs) have the non-trivial phases and the vacuum is not  $CP$  invariant even if Lagrangian is  $CP$  invariant. The simple model, in which  $CP$  violation can occur explicitly and spontaneously, is the two Higgs doublet model (THDM). Since the THDM induces large flavor changing neutral current (FCNC) in general, one imposes some additional symmetries such as discrete symmetry[4] or approximate global family symmetry[5] on the model.

SUSY models automatically avoid large FCNC because one Higgs doublet ( $H_1$ ) couples with down-sector and another ( $H_2$ ) couples with up-sector. Additional parameters such as soft SUSY breaking parameters could be the origin of  $CP$  violation and  $CP$  would be violated both explicitly and spontaneously in the SUSY models. The Higgs bosons are the most important particles which the experimentalists and theorists wait for observing. In this paper, we study the Higgs masses in the minimal and the next minimal supersymmetric standard model[6] (MSSM and NMSSM) with respect to the origin of  $CP$  violation. It is found that the masses strongly depend on whether  $CP$  is violated explicitly or spontaneously. And even in the explicit breaking case, the masses also depend on whether Higgs potential breaks  $CP$  symmetry or not.

Let us give the brief review of the MSSM and the NMSSM. If we take into account of only top yukawa coupling for the yukawa sector, the superpotential of the MSSM and the NMSSM are

$$W = h_t Q H_2 T^c + \mu H_1 H_2, \quad (1)$$

and

$$W = h_t Q H_2 T^c + \lambda N H_1 H_2 - \frac{k}{3} N^3, \quad (2)$$

respectively. Here  $H_1$  and  $H_2$  are Higgs doublet fields as

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}. \quad (3)$$

with

$$H_1 H_2 = H_1^0 H_2^0 - H_1^- H_2^+. \quad (4)$$

$Q$  is a third generation quark doublet superfield and  $T^c$  is a right-handed top quark superfield. Top yukawa coupling constant is denoted by  $h_t$ .  $N$  is a gauge singlet field. We neglect linear and quadratic terms of  $N$  because of imposing  $Z_3$  symmetry which might interpret weak scale baryogenesis[7]. If we include all these terms, the following discussion becomes quite different due to the additional parameters[8]. The parameters in the MSSM such as

$$\mu, \quad A_t, \quad B, \quad M_i \quad (i = 1, 2, 3), \quad (5)$$

are complex in general.  $A_t$  and  $B$  terms are soft SUSY breaking parameters corresponding to top yukawa coupling and  $\mu$  term, respectively.  $M_i$ s are gaugino mass parameters with the gauge group index  $i$ . By the  $R$  transformation and Higgs field redefinition, two complex phases among four phases in Eq.(5) can be rotated away. Then it is noticed that the  $CP$  phases other than KM phase exist in the MSSM[9]. As for the NMSSM, adding parameters

$$\lambda, \quad k, \quad A_\lambda, \quad A_k, \quad (6)$$

are also generally complex, where  $A_\lambda$  and  $A_k$  are soft SUSY breaking parameters corresponding to  $\lambda$  and  $k$  in Eq.(2). The NMSSM has more  $CP$  phases than the MSSM.

Section 2 is devoted to the explicit  $CP$  violation through the yukawa sector. In section 3, we discuss the explicit  $CP$  violation through the Higgs sector. In section 4, the spontaneous  $CP$  violation and the neutron electric dipole moment (NEDM) are analysed. Section 5 gives summary and discussion.

## 2 The explicit $CP$ violation through the "yukawa" sector

In general, explicit  $CP$  violation occurs through the yukawa sector and/or the Higgs sector. In the former scenario,  $CP$  violation should be induced by the yukawa couplings or scalar three point interactions, which we call "yukawa" sector  $CP$  violation, analysed in this section. The latter scenario, where the Higgs sector breaks  $CP$  symmetry explicitly, is discussed in section 3.

In this case, there is no mixing among scalar and pseudoscalar Higgs particles in the MSSM and NMSSM and the neutral Higgs mass matrix is

$$M_{H^0}^2 = \begin{pmatrix} \text{Re}[H_1, H_2, (N)] & \text{Im}[H_1, H_2, (N)] \\ \text{(scalar)} & 0 \\ 0 & \text{(pseudoscalar)} \end{pmatrix}. \quad (7)$$

One of the pseudoscalars which is the mixing state of  $H_1$  and  $H_2$  is the Goldstone boson absorbed by  $Z$  boson. There are two (three) neutral scalars and one (two) neutral pseudoscalar(s) in the MSSM (NMSSM) as physical particles.

It is well known that the one loop corrections have non-negligible effects on Higgs

masses in SUSY models[10]. The one loop effective potential[11] is

$$V_{1\text{-loop}} = \frac{1}{64\pi^2} \text{Str } M^4 \left( \ln \frac{M^2}{Q^2} \right), \quad (8)$$

where  $M$ s are the field dependent mass matrices. If we consider only top and stop contributions and also neglect the stop left-right mixing, Eq.(8) is reduced to be

$$V_{\text{top}} = \frac{3}{64\pi^2} \left[ (h_t^2 |H_2|^2 + m_t^2)^2 \ln \frac{(h_t^2 |H_2|^2 + m_t^2)}{Q^2} - h_t^4 |H_2|^4 \ln \frac{h_t^2 |H_2|^2}{Q^2} \right], \quad (9)$$

where  $m_{\tilde{t}}$  is the soft breaking stop mass.

By using Eq.(9), one can derive the upper bound of the lightest scalar masses both in the MSSM[10] and in the NMSSM[12][13][23] as

$$m_{h1} \leq M_Z^2 \cos^2 2\beta + \Delta v^2 \sin^4 \beta, \quad (10)$$

and

$$m_{h1} \leq M_Z^2 \cos^2 2\beta + \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \Delta v^2 \sin^4 \beta, \quad (11)$$

respectively. Here  $\Delta$  is defined as

$$\Delta \equiv \frac{3h_t^4}{4\pi^2} \ln \frac{m_{\tilde{t}}^2}{m_t^2}, \quad (12)$$

and VEVs of Higgs fields are

$$\langle H_1 \rangle = v_1, \quad \langle H_2 \rangle = v_2, \quad \langle N \rangle = x. \quad (13)$$

Here  $v_1, v_2$ , and  $x$  are real and positive parameters with  $v = \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV}$ .

We also define  $\tan \beta = v_2/v_1$ . The case of VEVs having non-vanishing relative phases is discussed later as the spontaneous  $CP$  violation scenario.

It is worth noting that Eqs.(10) and (11) do not change drastically by introducing more doublet- or singlet-Higgs fields[13].

Eq.(10) shows that the MSSM light Higgs mass becomes too small in the region  $\tan \beta \sim 1$ . At  $\tan \beta = 1$  first term is vanished and the loop effects play the essential

role to lift up the Higgs mass. However the situation is quite different in the NMSSM. The second term in the Eq.(11) still works around  $\tan\beta \sim 1$  and the scalar mass is not so small in contrast to the MSSM case. At large  $\tan\beta$ , the behaviors of scalar mass bound are almost the same in the MSSM and the NMSSM. It is also noticed that the larger stop mass becomes, the larger the light Higgs mass becomes from Eqs.(10) and (11). These behaviors are shown in Fig.1(a) and Fig.1(b). Through this paper we consider the case of  $\tan\beta \geq 1$ .

The lower bound of Higgs mass of the SM[14] is also shown in the Fig.1(a) and Fig.1(b). This lower bound is obtained from the SM vacuum stability and written as

$$m_{H_{\text{SM}}} > 132.0 + 2.2(m_t - 170.0) - \frac{4.5(\alpha_s - 0.117)}{0.007} \text{ (GeV)}. \quad (14)$$

Here we use  $\alpha_s = 0.129$ , which is the strong coupling constant at  $M_Z$  scale.

### 3 The explicit $CP$ violation through the Higgs sector

In this section, we discuss the case that there is explicit  $CP$  violation in the Higgs sector. Contrary to the previous situation Eq.(7), the neutral Higgs mass matrix becomes

$$M_{H^0}^2 = \begin{pmatrix} \text{Re}[H_1, H_2, (N)] & \text{Im}[H_1, H_2, (N)] \\ \text{(scalar)} & \sim \sin\phi \\ \sim \sin\phi & \text{(pseudoscalar)} \end{pmatrix}. \quad (15)$$

Here  $\phi$  is the phase that characterizes the  $CP$  violation in the Higgs sector.

In the MSSM, the tree level Higgs potential is automatically  $CP$  invariant.  $CP$  symmetry is violated by the radiative effects both explicitly and/or spontaneously. As for the spontaneous  $CP$  violation case, we will see in the next section. In the explicit  $CP$  violation case, the coefficients of  $\lambda_{5,6,7}$  in Ref.[2] derived by the radiative

corrections are relatively small such as  $\lambda_5 \simeq g^4/32\pi^2 \sim 10^{-4}$  due to the loop suppression factor[16]. The scalar-pseudoscalar mixing elements  $S_1$ - $A$  and  $S_2$ - $A$  of the neutral Higgs mass matrix are

$$\begin{aligned}
m_{S_1-A}^2 &= \cos \beta \operatorname{Im}(B\mu) + (\sin^3 \beta - 2 \cos^2 \beta \sin \beta) \operatorname{Im}(\lambda_5) v^2 \\
&\quad + (\cos \beta \sin^2 \beta - \frac{3}{2} \cos^3 \beta) \operatorname{Im}(\lambda_6) v^2 - \frac{1}{2} \cos \beta \sin^2 \beta \operatorname{Im}(\lambda_7) v^2, \\
m_{S_2-A}^2 &= -\sin \beta \operatorname{Im}(B\mu) + (2 \cos \beta \sin^2 \beta - \cos^3 \beta) \operatorname{Im}(\lambda_5) v^2 \\
&\quad + \frac{1}{2} \cos^2 \beta \sin \beta \operatorname{Im}(\lambda_6) v^2 + (\frac{3}{2} \sin^3 \beta - \cos^2 \beta \sin \beta) \operatorname{Im}(\lambda_7) v^2.
\end{aligned} \tag{16}$$

Here we can always take  $B\mu$  to be real by the Higgs field redefinition. Then the scalar-pseudoscalar mixing are so small that the situation becomes almost the same as the previous section. Then we go to the NMSSM following Ref.[15].

The scalar-potential of the NMSSM including top, stop loop effects by Eq.(9) is

$$V = V_{\text{nophase}} + V_{\text{phase}}, \tag{17}$$

where

$$\begin{aligned}
V_{\text{nophase}} &= |\lambda|^2[|H_1 H_2|^2 + |N|^2(|H_1|^2 + |H_2|^2)] + |k|^2|N|^4 \\
&\quad + \frac{g_1^2 + g_2^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g_2^2}{2}(|H_1|^2|H_2|^2 - |H_1 H_2|^2) \\
&\quad + m_{H_1}^2|H_1|^2 + m_{H_2}^2|H_2|^2 + m_N^2|N|^2 \\
&\quad + V_{\text{top}},
\end{aligned} \tag{18}$$

$$V_{\text{phase}} = -(\lambda k^* H_1 H_2 N^{*2} + \text{h.c.}) - (\lambda A_\lambda H_1 H_2 N + \text{h.c.}) - (\frac{k A_k}{3} N^3 + \text{h.c.}).$$

The parameters  $\lambda, k, A_\lambda$ , and  $A_k$  are all complex in general.  $CP$  phase cannot be included in the potential corrected by the loop effect  $V_{\text{top}}$ . So  $CP$  phase appears from only  $V_{\text{phase}}$  in Eq.(18). We can remove two complex phases by the field redefinition of  $N$  and  $H_1 H_2$ . So without loss of generality, we can take

$$\lambda A_\lambda > 0, \quad k A_k > 0. \tag{19}$$

Only one phase remains in  $\lambda k^*$  denoted as

$$\lambda k^* \equiv \lambda k e^{i\phi}. \quad (20)$$

Here  $\lambda$  and  $k$  on the right hand side are real and positive numbers. In addition to this phase, there appear  $CP$  phases from VEVs of  $H_1, H_2$ , and  $N$  in general. But now, we neglect these phases for simplicity and use Eq.(13). By using the three stationary conditions

$$\frac{\partial V}{\partial v_i} = 0 \ (i = 1, 2), \quad \frac{\partial V}{\partial x} = 0, \quad (21)$$

we can eliminate three parameters  $m_{H_1}^2, m_{H_2}^2$ , and  $m_N^2$ . Higgs fields are expanded around their minimum point as

$$\begin{aligned} H_1^0 &= v_1 + \frac{1}{\sqrt{2}}(S_1 + i \sin \beta A), \\ H_2^0 &= v_2 + \frac{1}{\sqrt{2}}(S_2 + i \cos \beta A), \\ N &= x + \frac{1}{\sqrt{2}}(X + iY). \end{aligned} \quad (22)$$

Here  $S_1, S_2$ , and  $X$  are scalars, and  $A$  and  $Y$  are pseudoscalars. By using this notation, we get  $5 \times 5$  neutral Higgs mass matrix as

$$M_{H^0}^2 = \begin{pmatrix} M_{S_1, S_2, X}^{S_1, S_2, X} & M_{S_1, S_2, X}^{A, Y} \\ (M_{S_1, S_2, X}^{A, X})^T & M_{A, Y}^{A, Y} \end{pmatrix}, \quad (23)$$

where  $M_{S_1, S_2, X}^{S_1, S_2, X}$ ,  $M_{S_1, S_2, X}^{A, Y}$ , and  $M_{A, Y}^{A, Y}$  are  $3 \times 3$ ,  $3 \times 2$ , and  $2 \times 2$  submatrices, respectively.

This matrix has the same form as Eq.(15). The matrix  $M_{S_1, S_2, X}^{S_1, S_2, X}$  of the scalar part of  $S_1, S_2$ , and  $X$  is

$$M_{S_1, S_2, X}^{S_1, S_2, X} = \begin{pmatrix} \bar{g}^2 v^2 \cos^2 \beta & (\lambda^2 - \bar{g}^2/2)v^2 \sin 2\beta & 2\lambda^2 v x \cos \beta \\ +\lambda x A_{\sigma_1} \tan \beta & -\lambda x A_{\sigma_1} & -\lambda v \sin \beta A_{\sigma_2} \\ (\lambda^2 - \bar{g}^2/2)v^2 \sin 2\beta & (\bar{g}^2 + \Delta)v^2 \sin^2 \beta & 2\lambda^2 v x \sin \beta \\ -\lambda x A_{\sigma_1} & +\lambda x A_{\sigma_1} / \tan \beta & -\lambda v \cos \beta A_{\sigma_2} \\ 2\lambda^2 v x \cos \beta & 2\lambda^2 v x \cos \beta & \frac{\lambda v^2}{2x} A_\lambda \sin 2\beta \\ -\lambda v \sin \beta A_{\sigma_2} & -\lambda v \cos \beta A_{\sigma_2} & -A_k k x + 4k^2 x^2 \end{pmatrix}, \quad (24)$$



where we define  $\bar{g}^2 \equiv (g^2 + g'^2)/2$ ,  $A_{\sigma_1} \equiv A_\lambda + kx \cos \phi$ , and  $A_{\sigma_2} \equiv A_\lambda + 2kx \cos \phi$ . The matrix  $M_{A,Y}^{A,Y}$  of the pseudoscalar part of  $A$  and  $Y$  is

$$M_{A,Y}^{A,Y} = \begin{pmatrix} 2\lambda x A_{\sigma_1} / \sin 2\beta & \lambda v A'_\sigma \\ \lambda v A'_\sigma & \frac{\lambda v^2}{2x} \sin 2\beta A_\lambda + 3A_k kx \\ & + 2\lambda k v^2 \sin 2\beta \cos \phi \end{pmatrix}, \quad (25)$$

where we define  $A'_\sigma \equiv A_\lambda - 2kx \cos \phi$ . The matrix  $M_{S_1,S_2,X}^{A,Y}$  of the scalar-pseudoscalar mixing part of  $A$  and  $Y$  is

$$M_{S_1,S_2,X}^{A,Y} = \begin{pmatrix} \lambda k x^2 \cos \beta \sin \phi & -2\lambda k v x \sin \beta \sin \phi \\ \lambda k x^2 \sin \beta \sin \phi & -2\lambda k v x \cos \beta \sin \phi \\ 2\lambda k v x \sin \phi & -\lambda k v^2 \sin 2\beta \sin \phi \end{pmatrix}. \quad (26)$$

At  $\phi = 0$ ,  $M_{S_1,S_2,X}^{A,Y}$  vanishes and the Higgs mass matrix reduces to the type of Eq.(7). So the light Higgs mass becomes the same as the one in the "yukawa" CP violation case and  $CP$  is conserved in the neutral Higgs sector.

Now we consider the large  $\tan \beta$  limit.  $S_1$ - $A$ ,  $S_2$ - $Y$ , and  $X$ - $Y$  components in the  $M_{S_1,S_2,X}^{A,Y}$  vanish at this limit. So it is enough to see the  $S_1$ - $Y$  and  $S_2$ - $X$ - $A$  submatrices. These are

$$\begin{pmatrix} \sim \lambda x A_{\sigma_1} \tan \beta & -2\lambda k v x \sin \phi \\ -2\lambda k v x \sin \phi & 3A_k kx \end{pmatrix}, \quad (27)$$

and

$$\begin{pmatrix} (\bar{g}^2 + \Delta)v^2 & 2\lambda^2 v x & \lambda k x^2 \sin \phi \\ 2\lambda^2 v x & -A_k kx + 4k^2 x^2 & 2\lambda k v x \sin \phi \\ \lambda k x^2 \sin \phi & 2\lambda k v x \sin \phi & 2\lambda x A_{\sigma_1} / \sin 2\beta \end{pmatrix}, \quad (28)$$

respectively. In each matrix, only  $S_1$ - $S_1$  and  $A$ - $A$  components are dominant and the scalar-pseudoscalar mixing is very small.  $CP$  violation in the neutral Higgs sector vanishes at the large  $\tan \beta$  limit. In this case, the light Higgs mass is the same as explicit  $CP$  violation in the "yukawa" sector.

The scalar-pseudoscalar mixing depends on the value of  $\tan \beta$ . In the region of  $\tan \beta \sim 1$ , this mixing becomes large. Then the light scalar mass become smaller

than the one without mixing. The  $\tan\beta$  dependence of the light scalar mass is shown in Fig.2, in which we take from Ref.[15], as

$$\begin{aligned} k &= 0.1, & \lambda &= 0.2, & m_{\tilde{t}} &= 3 \text{ TeV}, \\ A_k &= A_\lambda = v, & x &= 10 v. \end{aligned} \tag{29}$$

The Higgs particle gets smaller mass as the phase  $\phi$  becomes larger. In Fig.2, the following experimental constraints of Higgs search are considered.;

1. The lightest and the second lightest Higgs bosons denoted by  $h_1$  and  $h_2$  have not been observed in the decay of  $Z$ [17], so that  $Z \rightarrow h_1 + h_2$  should be forbidden kinematically. Then the condition

$$m_{h_1} + m_{h_2} > m_Z$$

is derived.

2. The lightest boson  $h_1$  has not been observed by the decay  $Z \rightarrow h_1 + Z^* \rightarrow h_1 + l^+ l^-$  [18]. The lower mass limit is

$$m_{h_1} > (65 \text{ GeV})(\alpha_1 \cos\beta + \alpha_2 \sin\beta)^2,$$

where  $\alpha_1$  ( $\alpha_2$ ) is the ratio of the  $S_1$  ( $S_2$ ) component of  $h_1$ .

3. The "pseudoscalar" boson should be larger than 22 GeV in the case of  $\tan\beta > 1$ [17].
4. In the MSSM, the lower limit of two Higgs scalars should be larger than 44 GeV in the case of  $\tan\beta > 1$ [17].

## 4 The spontaneous $CP$ violation

In this section, we discuss the spontaneous  $CP$  violation in the MSSM and the NMSSM. It occurs by the phase difference of VEVs of Higgs fields. As for the MSSM, the tree level Higgs potential is always  $CP$  invariant. However, as seen in the previous section, if the radiative corrections are included, there is the possibility of the spontaneous  $CP$  violation[16]. In this case, the light "pseudoscalar" appears with about 6 GeV mass and this is contradict with experiment[17]. In the NMSSM, if we consider only cubic couplings, the tree level potential cannot have  $CP$  violating vacuum[19]. However the one loop corrections could trigger spontaneous  $CP$  violation[20]. This scenario also demands the relatively light "pseudoscalar" compared to the no- $CP$  violation scenarios. The appearance of light particles in both the MSSM and the NMSSM is the general results by the Georgi-Pais theorem[21]. Since we study the possibility of the spontaneous  $CP$  violation, all parameters except for VEVs of  $H_1$ ,  $H_2$ , and  $N$  are assumed to be real. We obtain the Higgs mass around 50 GeV, which is compatible with the present experimental constraints, because there are adjustable parameters in the NMSSM.

Recently, Babu and Barr pointed out the possibility of the spontaneous  $CP$  violation in the NMSSM by using Eq.(9)[20]. The results of their analysis are summarized as

$$m_{h_1}^2 \leq CM_Z^2, \quad m_{h_1}^2 + m_{h_2}^2 \leq (C + \cos^2 \beta)M_Z^2, \quad (30)$$

where  $C \equiv [4A_\lambda(3A_\lambda - A_k)/A_k(4A_\lambda - A_k)](\lambda/\bar{g}^2)$ . The positivity of the mass eigenvalues and the experimental constraints limit the parameter to

$$1/3 \leq A_\lambda/A_k \leq 2.7. \quad (31)$$

So the upper bounds of Higgs masses are

$$m_{h_1} \leq 50 \text{ GeV}, \quad m_{h_1} + m_{h_2} \leq 100 \text{ GeV}. \quad (32)$$

The predictive charged Higgs mass  $m_{H^\pm} \leq 100 \text{ GeV}$  is not affected by the radiative corrections. As long as one uses Eq.(9),  $CP$  phase does not appear in Eq.(8). However, if stop left-right mixing terms are included, which are neglected in Eq.(9), there also exists the  $CP$  phase in the one loop level effective potential of Eq.(8). There is the possibility that the  $CP$  violating effects which appear in the one loop level might have the large effects on the Higgs masses. As for this, numerical analysis has been done in Ref.[22], where bottom and sbottom contributions are also included. We use the squark mass squared matrix  $M^2$  in Eq.(8) as

$$M_{\tilde{t}, \tilde{b}}^2 = \begin{pmatrix} \tilde{t}_L & \tilde{t}_R^c & \tilde{b}_L & \tilde{b}_R^c \\ m_{11}^2 & m_{12}^2 & m_{13}^2 & m_{14}^2 \\ m_{12}^{*2} & m_{22}^2 & m_{23}^2 & m_{24}^2 \\ m_{13}^{*2} & m_{23}^{*2} & m_{33}^2 & m_{34}^2 \\ m_{14}^{*2} & m_{24}^{*2} & m_{34}^{*2} & m_{44}^2 \end{pmatrix}, \quad (33)$$

where

$$\begin{aligned} m_{11}^2 &= m_Q^2 + h_t^2 |H_2^0|^2 + h_b^2 |H_1^-|^2 - \frac{g_1^2}{12} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2) \\ &\quad + \frac{g_2^2}{4} (|H_1^0|^2 - |H_1^-|^2 - |H_2^0|^2 + |H_2^+|^2), \\ m_{12}^2 &= h_t (A_t H_2^{0*} + \lambda N H_1^0), \\ m_{13}^2 &= -h_t^2 H_2^{0*} H_2^+ - h_b^2 H_1^{-*} H_1^0 + \frac{g_2^2}{2} (H_2^+ H_2^{0*} + H_1^{-*} H_1^0), \\ m_{14}^2 &= -h_b (\lambda N H_2^+ - A_b H_1^{-*}), \\ m_{22}^2 &= m_T^2 + h_t^2 (|H_2^0|^2 + |H_2^+|^2) + \frac{g_1^2}{3} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2), \\ m_{23}^2 &= h_t (\lambda N^* H_1^{-*} - A_t H_2^+), \\ m_{24}^2 &= h_t h_b (H_2^0 H_1^{-*} + H_2^+ H_1^{0*}), \end{aligned}$$

$$\begin{aligned}
m_{33}^2 &= m_Q^2 + h_b^2 |H_1^0|^2 + h_t^2 |H_2^+|^2 - \frac{g_1^2}{12} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2), \\
&+ \frac{g_2^2}{4} (-|H_1^0|^2 + |H_1^-|^2 + |H_2^0|^2 - |H_2^+|^2), \\
m_{34}^2 &= -h_b (A_b H_1^{0*} + \lambda N H_2^0), \\
m_{44}^2 &= m_B^2 + h_b^2 (|H_1^0|^2 + |H_1^-|^2) - \frac{g_1^2}{6} (|H_1^0|^2 + |H_1^-|^2 - |H_2^0|^2 - |H_2^+|^2).
\end{aligned}$$

Here the mass parameters  $m_Q, m_T$ , and  $m_B$  are the soft supersymmetry breaking squark masses, and the parameters  $A_t$  and  $A_b$  are the coefficients of the soft supersymmetry breaking terms as

$$V_{\text{soft}} = A_t h_t \tilde{t}_L \tilde{t}_R^* H_2^0 - A_b h_b \tilde{b}_L \tilde{b}_R^* H_1^0 + \text{h.c.} + \dots \quad (34)$$

By using Eq.(33), one loop contribution of the charged Higgs mass is obtained. And we can also analyze the large  $\tan\beta$  region, in which  $h_b \sim h_t$ . The brief review is shown as follows.

In the spontaneous  $CP$  violation case, there exists the phase difference of VEVs of Higgs fields in contrary to the Eq.(13). VEVs of Higgs fields are defined as

$$\langle H_1 \rangle = v_1 e^{i\varphi_1}, \quad \langle H_2 \rangle = v_2 e^{i\varphi_2}, \quad \langle N \rangle = x e^{i\varphi_3}. \quad (35)$$

We can always eliminate one phase by the field redefinition. So physical phases are two which are assigned as

$$\theta \equiv \varphi_1 + \varphi_2 + \varphi_3, \quad \delta \equiv 3\varphi_3. \quad (36)$$

The minimization conditions Eq.(21) are modified to

$$\frac{\partial V}{\partial v_i} = 0 \quad (i = 1, 2), \quad \frac{\partial V}{\partial x} = 0, \quad \frac{\partial V}{\partial \delta} = 0, \quad \frac{\partial V}{\partial \theta} = 0. \quad (37)$$

The soft breaking masses  $m_{H_1}^2, m_{H_2}^2$ , and  $m_N^2$  are eliminated by the stationary conditions of  $v_i$  and  $x$ . And  $\delta$  and  $k$  are eliminated by the stationary conditions of  $\delta$  and  $\theta$ , respectively.

In addition to the experimental constraints 1  $\sim$  4 in the previous section, we also impose the theoretical constraints[23]

$$|k| \leq 0.87, \quad |k| \leq 0.63. \quad (38)$$

They are derived by the assumption that perturbation of  $\lambda$  and  $k$  should remain valid up to the GUT scale. Considering all these constraints, we obtain Higgs masses as the curvatures of the potential at the minimum point. The input parameters are

$$\begin{aligned} \lambda &= 0.24, \quad m_{\tilde{t}_L, \tilde{b}_L} = 3 \text{ TeV}, \\ m_{\tilde{t}_R} &= 0.95 m_{\tilde{t}_L}, \quad m_{\tilde{b}_R} = 0.98 m_{\tilde{t}_L}, \\ A_t &= 1 \text{ TeV}, \quad A_b = 1.1 A_t, \\ A_k &= 20 v, \quad A_\lambda = 11 v, \quad x = 20 v, \end{aligned} \quad (39)$$

with the assumption of GUT scale universality[24]. Here,  $m_{\tilde{t}_L, \tilde{b}_L}$ ,  $m_{\tilde{t}_R}$ , and  $m_{\tilde{b}_R}$  are soft breaking masses.

In Ref.[22], we obtained the numerical results

$$m_{h_1} \leq 52 \text{ GeV}, \quad m_{h_1} + m_{h_2} \leq O(100 \text{ GeV}), \quad (40)$$

in compatible with the experimental constraints 1  $\sim$  4 given in the last section. These results are consistent with Ref.[20]. So we can say that the neutral Higgs masses are not largely influenced by the  $CP$  phase in the one loop potential. However, as for the charged Higgs mass, we obtain relatively large mass about 285 GeV by the full one loop corrections including stop and sbottom mass matrix in Eq(33). The charged Higgs has too large mass to be observed at LEP2 and this mass is enough large to be consistent with  $b \rightarrow s\gamma$  experiment[25].

We have found that the solution only exists around the region in which  $\tan\beta \simeq 1$ , squark soft breaking masses are about 3 TeV, and  $A_t$  and  $A_b$  are about 1 TeV.

At present the neutron electric dipole moment (NEDM) gives the important clue to check the various  $CP$  violation models beyond the SM. In the followings, we estimate the NEDM by using the parameters obtained in the spontaneous  $CP$  violation scenario. The chargino and the gluino contributions to the NEDM are shown in Fig.3. The non-vanishing phases appear from the chargino and the squark mass matrices even if all initial parameters are set to be real. We assume that the gaugino masses satisfy the GUT relations as

$$\frac{M_3}{g_s^2} = \frac{M_2}{g^2} = \frac{3}{5} \frac{M'}{g'^2}, \quad (41)$$

where  $M_3$ ,  $M_2$ , and  $M'$  are soft breaking masses associated with the  $SU(3)_C$ ,  $SU(2)_L$ , and  $U(1)_Y$  subgroups, respectively. In the following calculations, we assume that  $M_2$  is 1 TeV, sup and sdown masses are 3 TeV, and the flavor mixing is neglected for simplicity. By these assumptions, the chargino contribution (Fig.3 (a)) is larger than the gluino one (Fig.3 (b))[26]. The  $CP$  phase  $\theta$  appears from diagonalization of the chargino and the squark mass matrices, and we can not rotate away this phase by the field redefinition. For example, the EDM of down quark from the chargino contribution is

$$\begin{aligned} d_d/e &= \frac{\alpha_{em}}{4\pi \sin^2 \theta_W} \sin \theta \frac{\lambda x M_2 \tan \beta}{(m_{\omega_1}^2 - m_{\omega_2}^2)} \frac{m_d}{m_{\tilde{d}}^2} \\ &\times \sum_{i=1}^2 (-1)^i \left( -\frac{1}{3} I \left[ \frac{m_{\omega_i}^2}{m_{\tilde{d}}^2} \right] + J \left[ \frac{m_{\omega_i}^2}{m_{\tilde{d}}^2} \right] \right), \end{aligned} \quad (42)$$

where,

$$\begin{aligned} I[r] &\equiv \frac{1}{2(1-r)^2} \left( 1 + r + \frac{2r}{1-r} \ln r \right), \\ J[r] &\equiv \frac{1}{2(1-r)^2} \left( 3 - r + \frac{2r}{1-r} \ln r \right), \end{aligned} \quad (43)$$

and  $\omega_i$ s are the chargino mass eigen-states. The gluino contribution is also estimated by the same way. The chargino and the gluino contributions are shown in Fig.4 by

using the non-relativistic relation  $d_n = (4d_d - d_u)/3$ . Spontaneous CP violation in the NMSSM predicts  $d_n \simeq O(10^{-26})$  e·cm which is small at one order compared to the present experimental upper limit of the NEDM[17].

## 5 Summary and Discussion

We have studied the Higgs masses in the context with the  $CP$  violation structures in the MSSM and the NMSSM. In the explicit  $CP$  violation, the lightest Higgs mass can be large about  $130 \sim 150$  GeV in both the MSSM and the NMSSM. This results do not change drastically whether  $CP$  violation exists Higgs sector or not. We have shown that the problem of the spontaneous  $CP$  violation scenario in the MSSM, which requires the light Higgs mass around a few GeV, is solved by extending the MSSM into the NMSSM with the singlet superfield. We also predict the NEDM which is smaller in one order than the present experimental upper limit. In this scenario, the lightest Higgs mass is  $m_{h_1} \leq 52$  GeV and sum of two light Higgs masses should be around 100 GeV. It is expected that LEP2 experiment will give the answer for the possibility of spontaneous  $CP$  violation in the Higgs sector.

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## Figure Captions

**Fig.1** The upper/lower bounds of Higgs masses versus  $\tan\beta$ . The parameter  $\lambda = 0.4$  is fixed. Solid line: the upper bound of the lightest scalar mass in the NMSSM; long-dashed-dotted line: the upper bound of the lightest scalar mass in the MSSM; dashed line: the lower bound of Higgs mass in the SM;

Fig.1(a):  $m_{\tilde{t}} = 1$  TeV;

Fig.1(b):  $m_{\tilde{t}} = 3$  TeV.

**Fig.2** The NMSSM lightest Higgs mass versus  $\tan\beta$  with the explicit  $CP$  violation in the Higgs potential. Solid line:  $\phi = \pi/2$ ; dashed line:  $\phi = \pi/4$ ; long-dashed-dotted line:  $\phi = 0$ .

**Fig.3** The diagram which contribute to the NEDM.

(a): The chargino contribution; (b): the gluino contribution.

**Fig.4(a)** The dependence of the NEDM on the phase  $\theta$  at  $\tan\beta = 1$ . The region of  $\theta$  is where the spontaneous  $CP$  violation in the NMSSM is available[22]. Solid line: the chargino contribution; long-dashed-dotted line: the gluino contribution; dashed line: the experimental upper limit  $11 \times 10^{-26}$  e·cm.

**Fig.4(b)** The  $x = \langle N \rangle$  dependence of the NEDM at  $\theta = 1.7$ . The region of  $x$  is where the spontaneous  $CP$  violation in the NMSSM is available. Solid line: the chargino contribution; long-dashed-dotted line: the gluino contribution; dashed line: the experimental upper limit.

**Table**      The light Higgs masses for models (MSSM or NMSSM) with origins of  $CP$  violation.

| Model | $CP$ violation | Mass bound |                                  |  |
|-------|----------------|------------|----------------------------------|--|
| MSSM  | explicit       | "yukawa"   | $m_{h_1}$                        | $\leq M_Z^2 \cos^2 2\beta + \Delta v^2 \sin^4 \beta$<br>$\leq 130 \sim 160 \text{ GeV}$ (small at $\tan\beta \sim 1$ )           |
|       |                | Higgs      |                                  | Almost same as "yukawa" case   |
|       | spontaneous    |            | $m_{h_1}$                        | $\leq 6 \text{ GeV}$ (excluded from experiment)  |
| NMSSM | explicit       | "yukawa"   | $m_{h_1}$                        | $\leq M_Z^2 \cos^2 2\beta + \frac{1}{2}\lambda^2 v^2 \sin^2 2\beta + \Delta v^2 \sin^4 \beta$<br>$\leq 130 \sim 160 \text{ GeV}$ |
|       |                | Higgs      | $m_{h_1}$                        | $\leq O(150 \text{ GeV})$  |
|       | spontaneous    |            | $m_{h_1}$<br>$m_{h_1} + m_{h_2}$ | $\leq 52 \text{ GeV}$<br>$\leq O(100 \text{ GeV})$   |

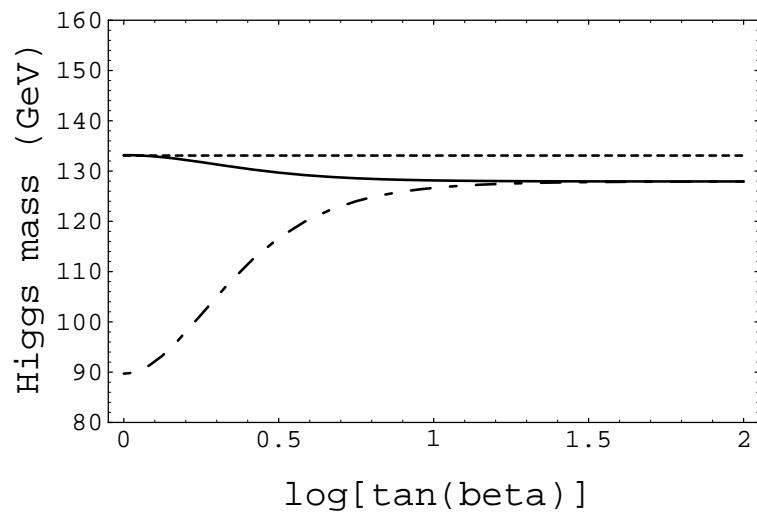


Fig.1 (a)

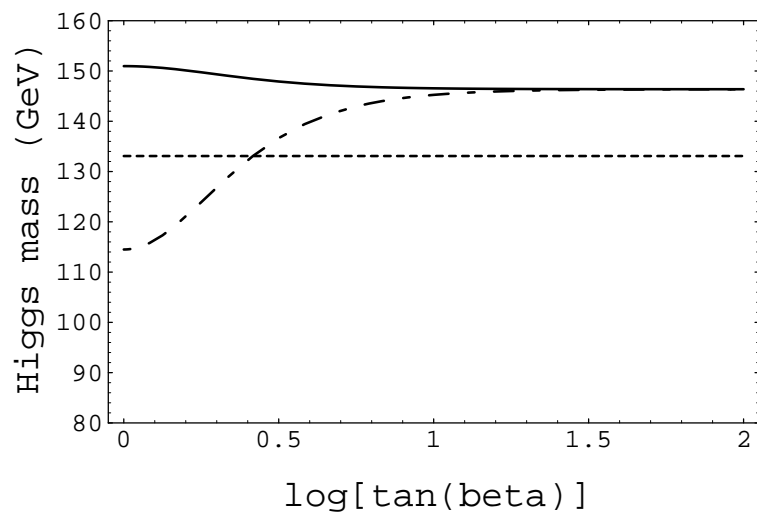


Fig.1 (b)

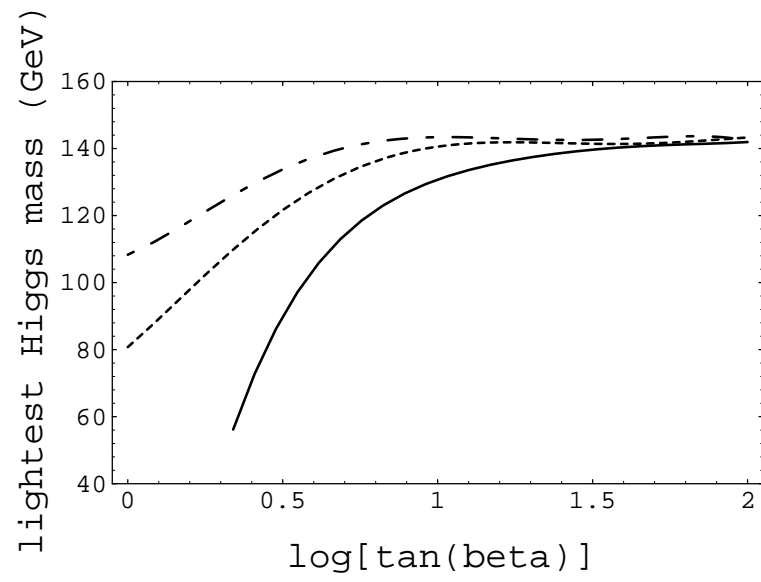


Fig.2

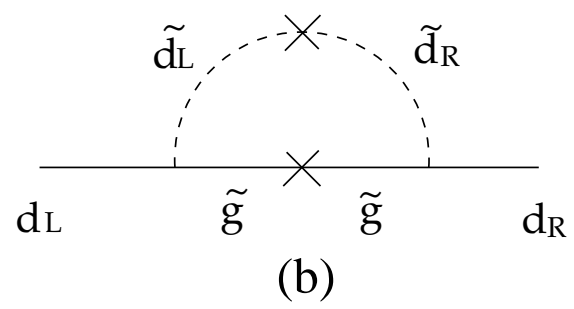
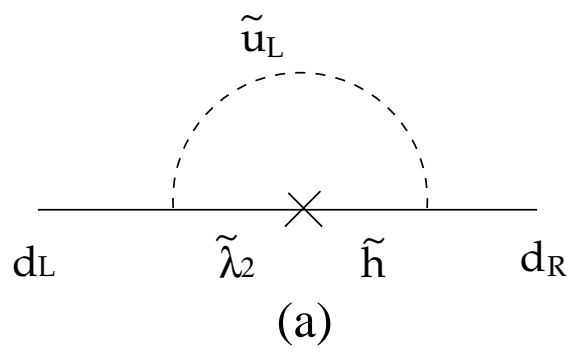


Fig.3



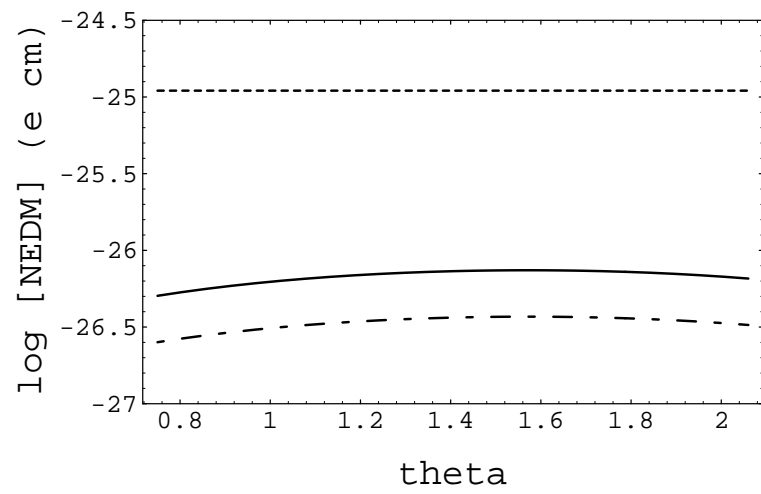


Fig.4 (a)

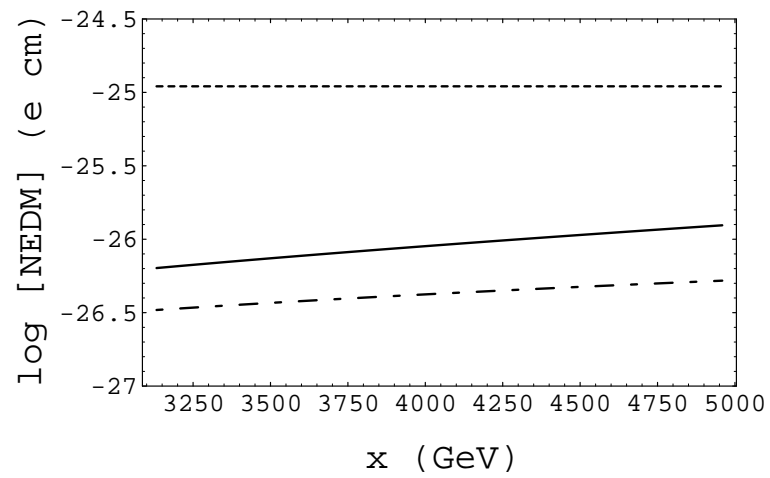


Fig.4 (b)